White Paper

Square Function Approximation to Estimating Inhalation Intakes

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Background

For General Steel Industries, NIOSH proposed estimating inhalation intakes using the 95th percentile of the distribution of surrogate airborne data collected while handling uranium metal. In the NIOSH estimate, the airborne material was assumed to cause an intake only during the time individuals were present and working with uranium metal. Resuspension of uranium contamination was accounted for separately.

During GSI work group deliberations, SC&A raised a concern that the airborne activity does not diminish instantly when the metal operations end and, therefore, the NIOSH approach could underestimate the intakes received by workers. This paper is intended to explore this issue.

Equations Describing Airborne Activity

The airborne active created from any operation can be described in terms of the production and removal mechanisms for the airborne material. The rate of change of the airborne activity can be described as the production rate minus the removal rate as described in the equation below:

$$\frac{dA}{dt} = P - \lambda A$$

where A is the airborne activity, P is the production rate and λ is the removal rate. For multiple removal mechanisms, λ represents the sum of the removal rates. The air concentration at any time after the beginning of operations can be expressed as:

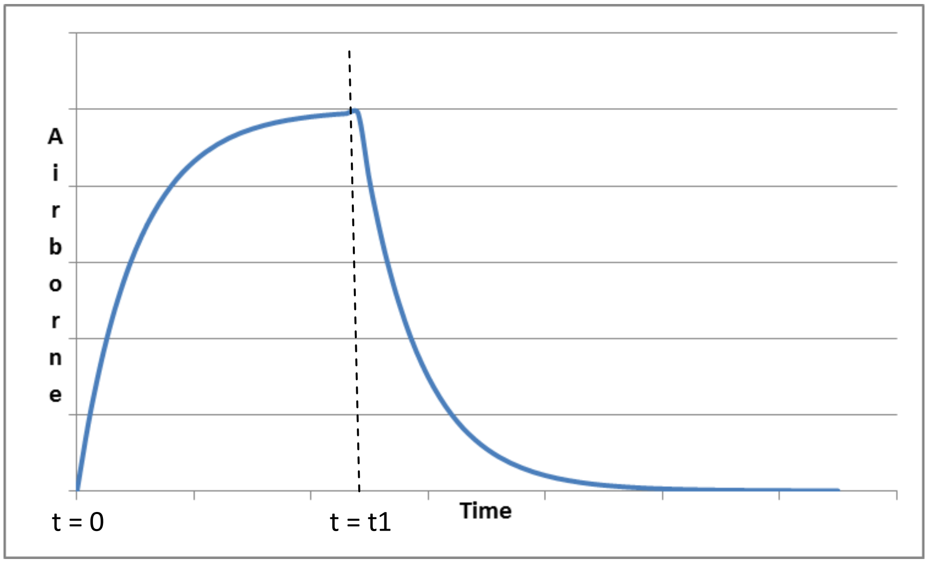
$$A = \frac{P}{\lambda} * \{1 - \exp(-\lambda t)\}$$

It can be seen from each of these equations that, without a removal mechanism, the airborne activity will continue to rise as long as there is a production term. In reality, there are removal mechanisms and the airborne activity will reach a maximum value equal to P/ λ, when the removal rate equals the production rate. Also, from these equations, it can be seen that while the level of this maximum activity is dependent on both the production rate and the removal rate, the time necessary to reach the maximum activity is driven solely by the removal rate.

Single Episode

In the simple case of a single episode of airborne generating activity, the airborne concentration will increase during the operation (up to a maximum) and decrease after the cessation of the operation. Figure 1 depicts this effect with the airborne generating activity occurring between time zero and time t1.

Figure 1 – Buildup and Decline of Airborne Activity from a Single Episode

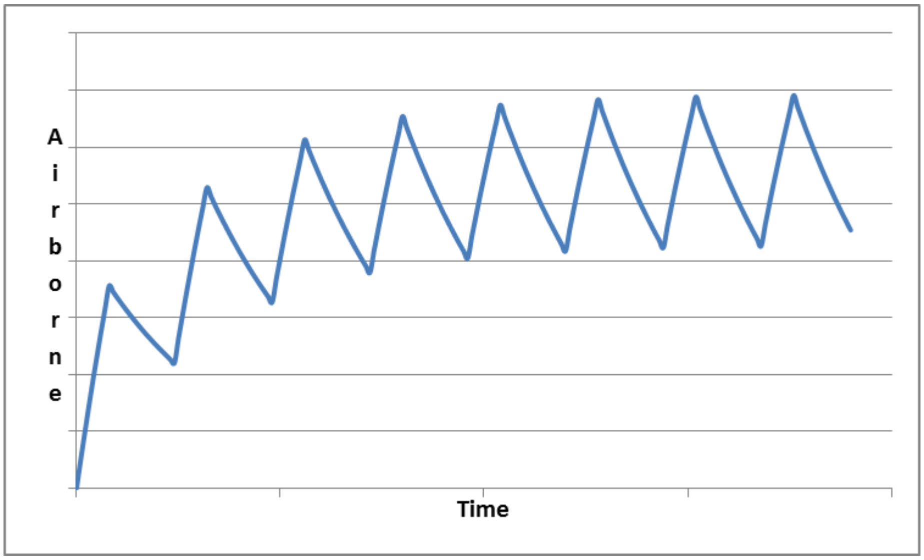


In Attachment A, the equations necessary to describe this curve are shown, as well as the mathematics associated with integrating the curves over time from time equal to zero to time equal to infinity. The resulting area is equal to P/λ*t1. Realizing that P/λ is the maximum airborne activity for continuous operations, this area is equivalent to assigning that airborne activity only for the duration of the operation. This demonstrates that any intakes received while the airborne activity is decreasing (after operations) are accounted for by overestimating the intake during the buildup portion of the curve.

Intermittent Episodes

If, after the first airborne causing episode, a second episode begins, the airborne activity in the area will include both sources. These would have an additive affect and cause the airborne activity during the second episode to increase to higher levels than experienced in the first episode. Figure 2 shows a graph depicting this effect.

Figure 2 – Buildup and Decline of Airborne Activity from a Multiple Episodes

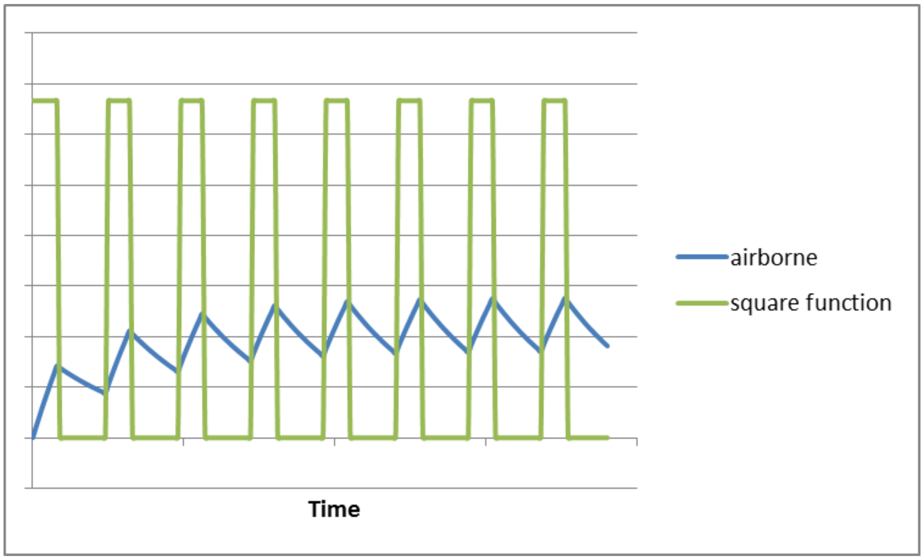


The airborne activity from multiple events can be calculated by adding the overlapping times of multiple single events. Since Attachment A demonstrates that an intake from a single event can be calculated using the square function, it follows that the intake from multiple events can be estimated using multiple square functions. The total area of multiple square functions would be the airborne activity multiplied by the total time of operation.

Intakes

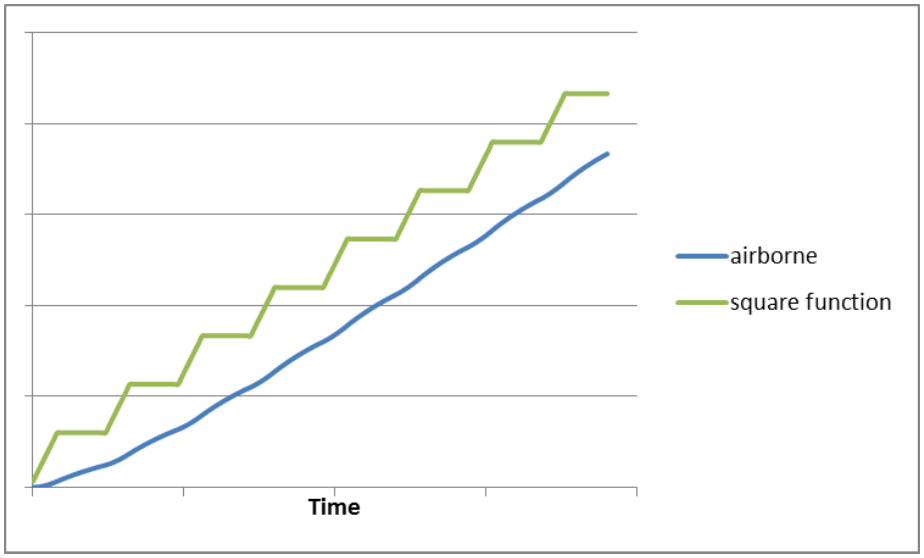
Attachment A provides the mathematical basis for indicating the square function is a good estimate of the intakes. For illustrative purposes, Figure 3 below superimposes that function on the saw toothed function above.

Figure 3 – Airborne Activity from a Multiple Episodes with Square Function Superimposed



From the figure, it is not easily seen that the area under both curves is equal. In reality, they are not. They are only equal if the saw toothed curve is extended out to infinity. Until then, the square function is higher. It is more illustrative to show a graph of the area under the curves (Figure 4).

Figure 4 – Integrated Airborne Activity from Multiple Episodes



While Attachment A indicated the area under both curves is equal, this graph shows the square function is actually higher. The reason for this is the equations in Attachment A were integrated to infinity.

Figures 5 and 6 below show Figures 3 and 4 extended out further in time after the end of intermittent operations.

Figure 5 – Figure 3 with Extended Time

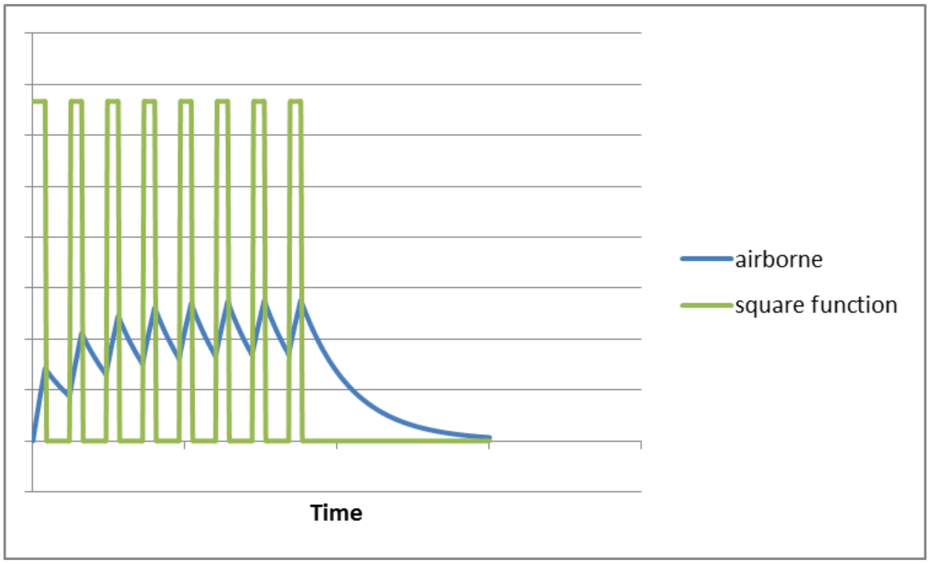
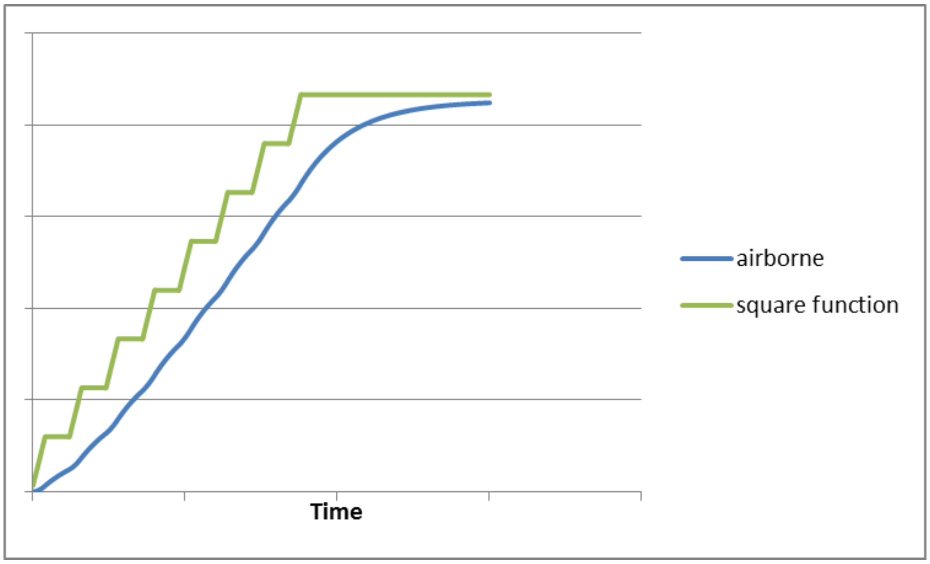


Figure 6 – Figure 4 with Extended Time



From these graphs, it can be seen that the area under the two curves will eventually be equal but the square function reaches the final value faster.

Effect of Different Removal Rates

In the graphs presented so far in this report, the values of parameters were chosen to illustrate the dynamics of the airborne activity. This leads to the question of what would the effect be of different values for the removal rate. All the graphs (except Figures 5 and 6) show a 190 hour time period with a removal rate of 0.03 hours. Below are graphs showing various removal rates. It is clear from these graphs that a fast removal rate would closely resemble the square function. This makes intuitive sense because a fast removal rate would cause the airborne activity to reach an equilibrium value quickly after the start of operations and decrease to zero quickly after operations. That pattern would closely mimic the square function. For slower removal rates, the square function over-predicts the airborne even more during the operations.

Figure 7 – Integrated Airborne with λ = 1

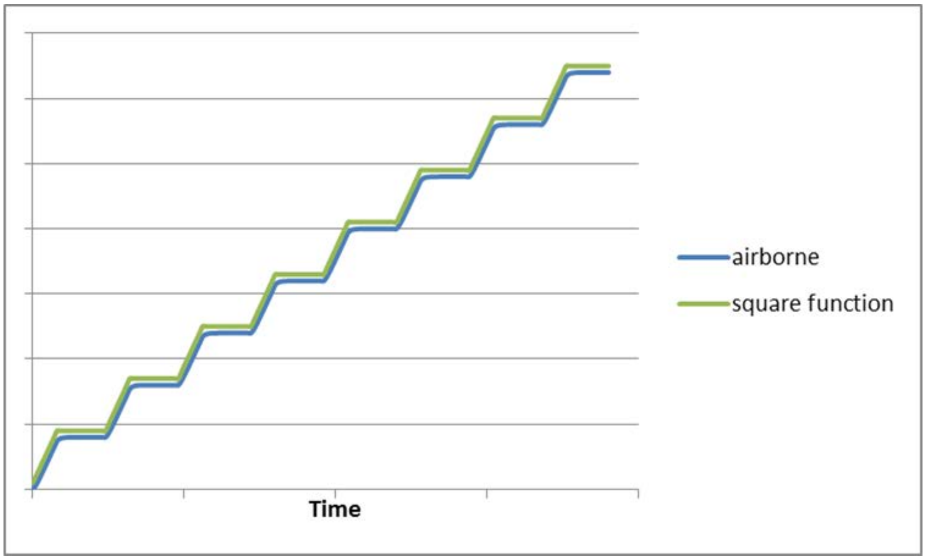


Figure 8 – Integrated Airborne with λ = 0.1

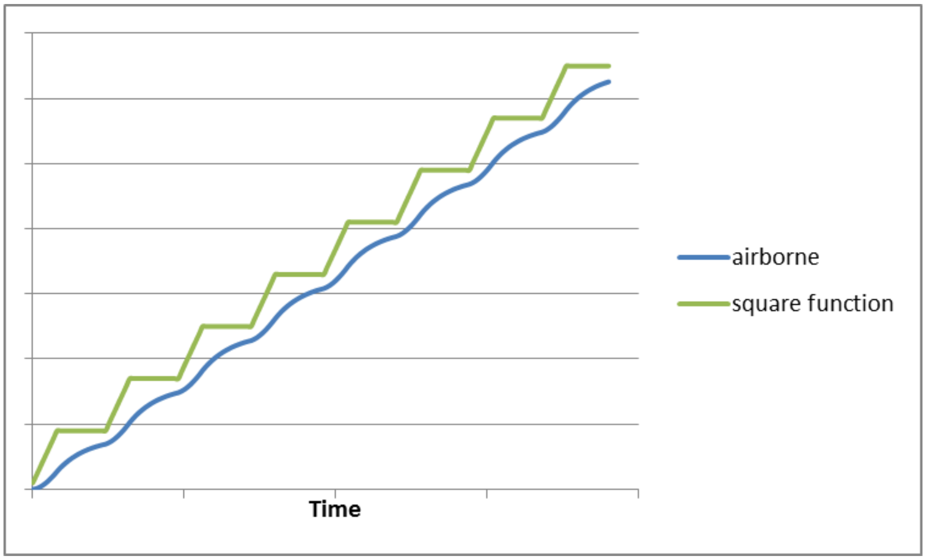


Figure 9 – Integrated Airborne with λ = 0.01

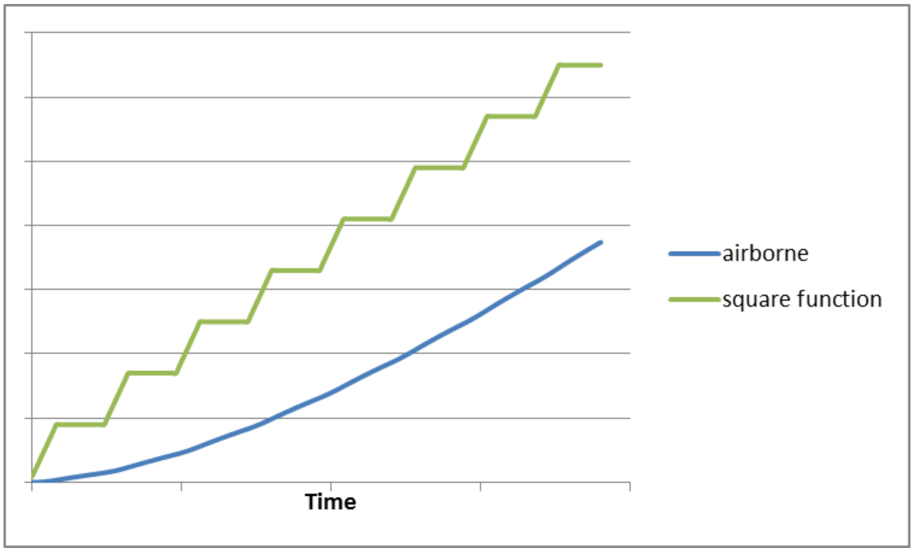
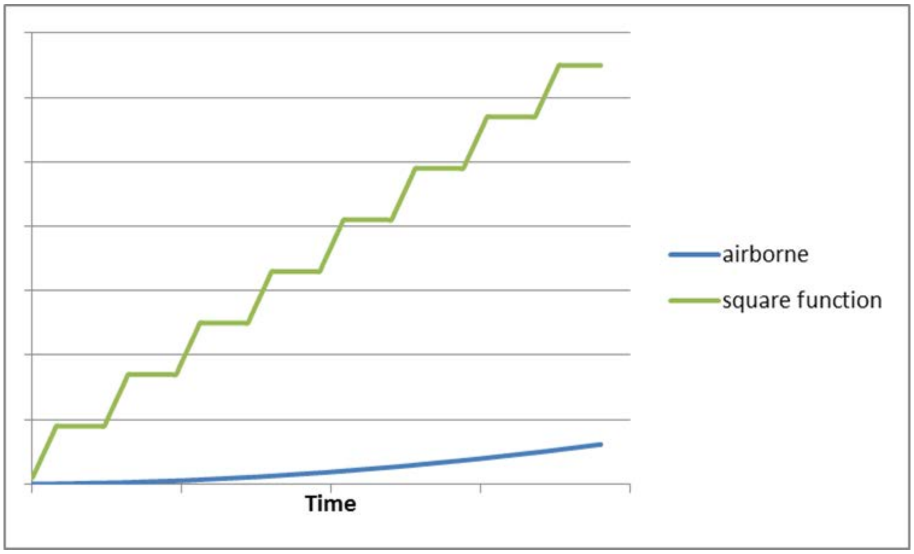


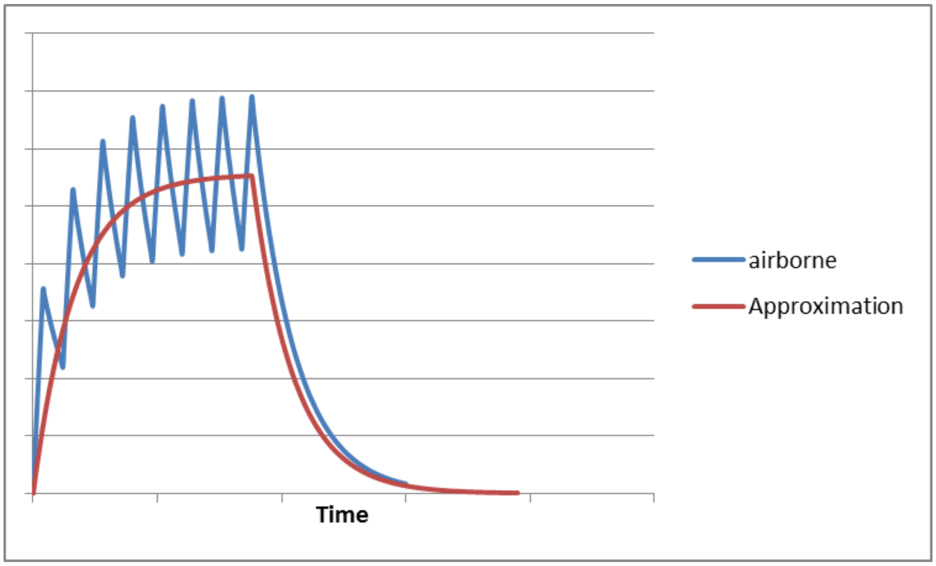
Figure 10 – Integrated Airborne with λ = 0.001



Approximation

An alternative method of estimating intakes would be to assume a continuous intake with the production rate prorated to account for the fraction of time operations occurred. For example, if operations occurred 8 hours per day every day, the production rate would be multiplied by 1/3 (8 hours/24 hours). An example of this effect is shown in the figure below over the extended time frame.

Figure 3 – Airborne Activity from Multiple Episodes with Approximation Superimposed



This approach would use the same equations in Attachment A, but in this case, "P" is replaced with "fP" where f is the prorating factor. From Attachment A, this would result in the integrated airborne activity being described by $fP/\lambda * t1$. This would be equivalent to assuming the airborne concentration (reduced by f) being applied only for the duration of the operations. In this case, the duration of operations would be continuously over the entire time frame of intermittent operations.

This analysis implies that any airborne activity applied throughout a period of intermittent operations must be prorated for the amount of time actual operations occurred.

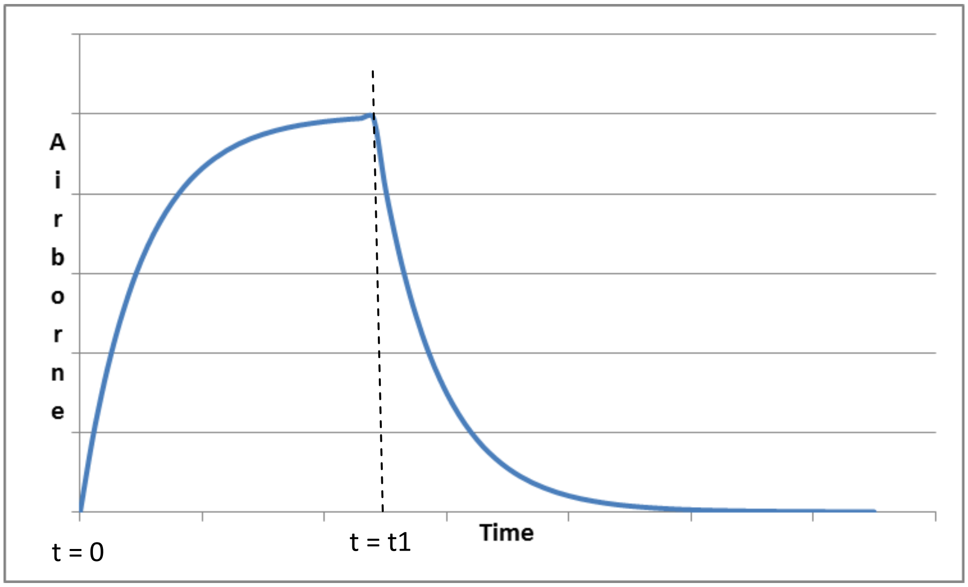
Conclusion

Intakes from airborne activity can be shown mathematically to be accounted for using the square function. The square function simply uses the airborne activity times the length of time the airborne causing evolutions occur. The square function overestimates the intakes at the start of the operation and underestimates after the end of the operation. The two effects mathematically cancel. Exploration of multiple episodes indicates the same is true for multiple episodes. Exploration of various removal rates also shows the same results.

It is also important to note that this analysis assumes someone is always present in the area. Further adjustments should normally be made for period of time when an individual leaves the area.

Attachment A
Mathematical Comparison of Buildup and Decay to Square Function

Figure A-1 – Building up and Decline of Airborne Activity from a Single Episode



The graph above shows the buildup and decline of airborne activity from a single airborne generating operation starting at time equal to zero and ending at time equal to t1. The buildup can be described by equation 2 (reiterated here).

$$A = \frac{P}{\lambda} * \{1 - \exp(-\lambda t)\}$$

The airborne level at t1 is then described by replacing t in the equation above with t1. At that point, that airborne activity begins to decline at a rate described by:

$$A = \frac{P}{\lambda} * \{1 - \exp(-\lambda t1)\} * \exp(-\lambda * (t - t1))$$

For someone continuously in the area, the intake can be described as the area under the curve which is the integration of the two equations above over the appropriate time periods:

$$area = \int_0^{t1} \frac{P}{\lambda} \{1 - \exp(-\lambda t)\, dt + \int_{t1}^{\infty} \frac{P}{\lambda} \{1 - \exp(-\lambda t1)\} * \exp(-\lambda (t - t1))\, dt$$

Noting that in the last term:

$$\exp(-\lambda t1) * \exp\big(-\lambda(t - t1)\big) = \exp(-\lambda t1) * \exp(-\lambda t) * \exp(+\lambda t1) = \exp(-\lambda t)$$

The equation for the area can be expanded to

$$area = \frac{P}{\lambda}\int_0^{t1} dt - \frac{P}{\lambda}\int_0^{t1} \exp(-\lambda t)\,dt + \frac{P}{\lambda}\exp(+\lambda t1)\int_{t1}^{\infty} \exp(-\lambda t)\,dt - \frac{P}{\lambda}\int_{t1}^{\infty} \exp(-\lambda t)\,dt$$

This can be solved as:

$$area = \frac{P}{\lambda}t1 - \frac{P}{\lambda^2}\{1 - \exp(-\lambda t1)\} + \frac{P}{\lambda^2}\exp(+\lambda t1)\exp(-\lambda t1) - \frac{P}{\lambda^2}\exp(-\lambda t1)$$

Noting that:

$$\exp(+\lambda t1)\exp(-\lambda t1) = 1$$

This simplifies to:

$$area = \frac{P}{\lambda}t1 - \frac{P}{\lambda^2}\{1 - \exp(-\lambda t1)\} + \frac{P}{\lambda^2}\{1 - \exp(-\lambda t1)\}$$

or

$$area = \frac{P}{\lambda}t1$$

Note that P/λ is the equilibrium airborne activity and t1 is the time the airborne activity was produced. This demonstrates that for an individual present in the area the entire time, the intake can be estimated by assuming the equilibrium activity was present from the start but only for the duration of the airborne causing evolution.